

Use of the CD matrix keywords in SIRTf images

May 14, 2003

Some changes are being made to the World Coordinate System (WCS) keywords in the Basic Calibrated Data (BCD) FITS files for IRAC and MIPS images. This document describes what new keywords will be written to the FITS headers, the reasons the change is being made, the meaning of the new keywords, and their relation to the old keywords. To ease the transition to these keywords, software that supports them is identified, and transformations are also outlined.

The intended audience for this document includes Legacy and GTO teams, and other groups, who are already working with BCD header information for IRAC and MIPS images. Nothing in this document applies to IRS spectra, or to MIPS SED mode data.

1 Changes to WCS keywords

Near the end of the section of the BCD header under “TARGET AND POINTING INFORMATION”, the keywords CDELTA1, CDELTA2, and CROTA2 will be deleted. In their place will appear the four CD-matrix keywords, CD1_1, CD1_2, CD2_1, and CD2_2; plus keywords tentatively named PXSCAL1, PXSCAL2, and PA. The meaning of these keywords are addressed in the next section. The last three keywords are intended for the convenience of human readers only.

In addition, under the header section “DISTORTION KEYWORDS”, the linear keywords A_0_1, A_1_0, B_0_1, and B_1_0 will either be omitted or have values equal to zero.

Pointing refinement keywords will change slightly. For details, see Section 3.3.

To summarize, the main change is that these keywords in the old scheme:

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CDELTA1 =          0.0003333333 / [deg/pix] Plate scale for axis 1 at CRPIX1,CRPIX2
CDELTA2 =          0.0003333333 / [deg/pix] Plate scale for axis 2 at CRPIX1,CRPIX2
CROTA2  =      83.3153059398739 / [deg] Position angle of axis 2 (W of N)
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are replaced by the following keywords in the new scheme (note that names PA, PXSCAL1, PXSCAL2, and the text in the comments, are tentative):

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CD1_1 = 3.655556841E-05 / [deg/pix] 1,1 element of CD matrix
CD1_2 = -3.328246759E-04 / [deg/pix] 1,2 element of CD matrix
CD2_1 = 3.351534900E-04 / [deg/pix] 2,1 element of CD matrix
CD2_2 = 3.900777902E-05 / [deg/pix] 2,2 element of CD matrix
PXSCAL1 = 1.21363 / [arcsec/pix] Axis 1 scale at CRPIX1,CRPIX2
PXSCAL2 = 1.20637 / [arcsec/pix] Axis 2 scale at CRPIX1,CRPIX2
PA = -83.3153059398739 / [deg] Position angle of axis 2 (E of N)

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2 Why this change is being made

To better align our products with the WCS standard. The first part of the WCS standard became effective in December 2002. This standard is described in Greisen & Calabretta (2002; hereafter Paper I) and Calabretta & Greisen (2002; Paper II). Matrix forms of the translation from pixel coordinates to the plane of projection are now standard, and the use of CROTA2 is deprecated. Furthermore, in April 2002, these authors reviewed the SSC’s distortion representation and made it clear that mixing “old-fashioned” CROTA2 with new keywords for distortion would be at odds with the standard.

To prevent scale and rotation changes from being “hidden” in the distortion polynomials. Limiting the distortion polynomials to quadratic-and-higher terms in the pixel-to-sky direction allows scale, rotation, and skew at the reference point to be unambiguously determined from the CD matrix elements, without having to inspect the distortion polynomial coefficients. If linear terms are allowed in the distortion polynomials, great care must be taken to avoid hiding a scale change or rotation offset within.

To reduce ambiguities over definitions and conventions. CROTA2 has never had a formal definition. A convention has become widely encoded in software, but the “West of North” sense confuses many people. Furthermore, when distortion is introduced into a transformation scheme, the meaning of a bulk rotation becomes even more obscure, since the pixel axes are not uniformly straight, nor exactly perpendicular when mapped to the plane of projection. By contrast, the transformation represented by CD-matrix terms coupled with quadratic-and-higher distortion terms is unique and unambiguous.

To ease translation between alternative distortion representations. For example, it should be possible to translate the TAN-SIP scheme for SIRTf to the ‘TNX’ distorted tangent representation in IRAF. Similarly, should the work on standardizing distortion representations in “Paper IV” by Calabretta et al (2003) result in widespread adoption, the SIRTf TAN-SIP form in principle could be translated to one of those forms. As another example, the matrix form for distortions used by the SIRTf Focal Plane Survey may be readily recast in our new scheme by separating linear and quadratic terms. All of these translations are eased by using the CD-matrix form.

3 Details on the changed keywords

3.1 Meaning of the WCS keywords

Following Papers I & II, we denote “intermediate world coordinates” in the plane of projection as object angles x, y , in units of degrees aligned with the world coordinate system. The origin of x, y is at the celestial coordinates represented by CRVAL1,CRVAL2.

We define u, v to be relative pixel coordinates with origin at CRPIX1,CRPIX2 (same origin as x, y). Let $f(u, v)$ and $g(u, v)$ be the quadratic and higher-order terms of the distortion polynomial. Then

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} CD_{11} & CD_{12} \\ CD_{21} & CD_{22} \end{pmatrix} \begin{pmatrix} u + f(u, v) \\ v + g(u, v) \end{pmatrix} \quad (1)$$

Let A_{pq} and B_{pq} be the polynomial coefficients for polynomial terms $u^p v^q$. Then for cubic distortion

$$f(u, v) = A_{20}u^2 + A_{02}v^2 + A_{11}uv + A_{21}u^2v + A_{12}uv^2 + A_{30}u^3 + A_{03}v^3 \quad (2)$$

$$g(u, v) = B_{20}u^2 + B_{02}v^2 + B_{11}uv + B_{21}u^2v + B_{12}uv^2 + B_{30}u^3 + B_{03}v^3 \quad (3)$$

The CD-matrix values together with the higher-order distortion polynomials, as in Equations 1,2, and 3, define a unique transformation from pixel coordinates to the plane-of-projection. This is illustrated graphically in Figure 1.

A legal (i.e. non-singular) 2-dimensional CD matrix may be deconstructed into separate steps of scaling, skew, and rotation, applied in that order. Details are given in the Appendix.

Both the new and old schemes include polynomial coefficients for the reverse direction, in which linear terms are allowed. These polynomials are applied to the the product of the inverse of the CD matrix with x, y . They are included only for efficiency, to avoid a potentially time-consuming iterative solution.

3.2 The additional keywords PA, PXSCAL1, and PXSCAL2

The three keywords PA, PXSCAL1, and PXSCAL2 are included for the convenience of human readers, and contain the pixel scales at the reference point (CRPIX1,CRPIX2—usually the array center) in units of arcseconds, and the approximate position angle of pixel axis 2 at the reference point in units of degrees, with the traditional E of N convention. The value of PA is written at pointing transfer time. *It is recommended that software use the CD matrix and distortion keywords for proper calculations.*

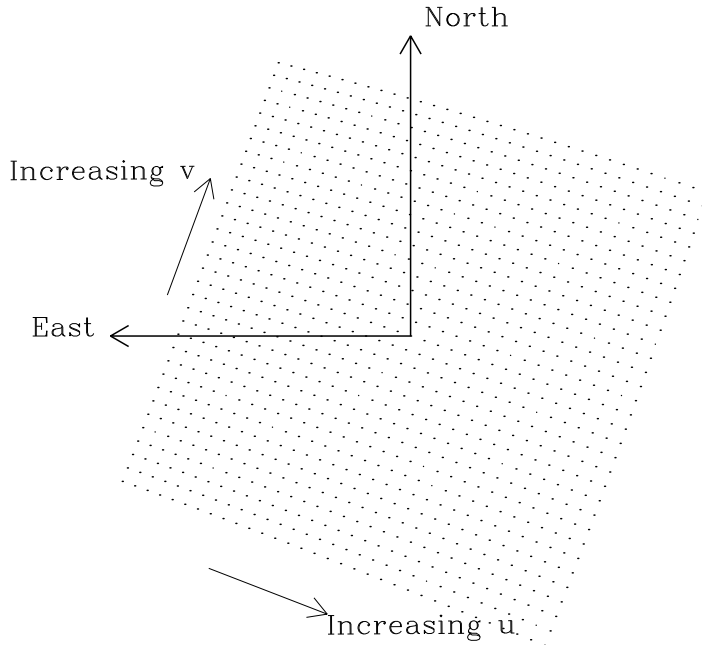


Figure 1: The dots are the corners of pixels for the $70\mu\text{m}$ array, fine scale, mapped onto the plane-of-projection (e.g. a tangent plane), based on polynomial coefficients from Morrison & Stamper (2003). The plane-of-projection is aligned with the celestial coordinate system (“North” and “East”), with origin at celestial coordinates (CRVAL1,CRVAL2), and is otherwise independent of the pixel coordinates. Relative pixel coordinates u, v have origin at FITS pixel coordinates (CRPIX1,CRPIX2), corresponding to the plane-of-projection origin, and are otherwise defined without reference to the sky. In this case, the dots have u and v values that are signed integers in the range $[-16, 16]$. The CD-matrix plus higher distortion terms is simply the unique polynomial transformation from relative pixel coordinates to the plane-of-projection.

3.3 Pointing refinement keywords

In the old scheme, the pointing refinement module in the SSC's post-BCD software outputs a refined value for CROTA2 as CT2RFND, as well as its uncertainty. We expect to write out refined CD-matrix keywords (with different names than CD1_1 etc.) as well as a refined analog to PA. Details are TBD as of this writing.

4 Answers to frequently asked questions

Why can't CROTA2, CDELTA1, and CDELTA2 be written to the headers along with the CD-matrix keywords?

The main reason why these keywords will not be written is that any software that used these keywords in place of the CD-matrix keywords will obtain the wrong results, due mainly to the lack of a linear skew term. A secondary reason is that mixing the deprecated CROTA2 with new distortion keywords runs counter to the sense of Paper II.

Mosaics are planned to be written with CROTA2. This is fine because no new WCS keywords are used, and no distortion is present.

What software will support the new scheme?

The SSC's post-BCD software is being upgraded to allow input of the new scheme (CD matrix support) in addition to the old.

Doug Mink has implemented SIRTf distortion support in his WCS routines, that will handle either old or new scheme. See version 1.33.2 or later at <http://tdc-www.harvard.edu/software/wcstools> . SAOimage (v 1.33.2 and later) uses these routines and will handle the distortions. The Montage software (v 1.6 or later) also uses Doug's routines and will handle either scheme. Doug has passed his routines to the developer of DS9, and we hope that program will be upgraded shortly.

5 References

Calabretta, M.R., & Greisen, E.W. 2002, A&A, 395, 1077 (Paper II).

Calabretta, M.R. et al., 2003, draft of Paper IV at <http://www.atnf.csiro.au/people/mcalabre/WCS/index.html> .

Greisen, E.W., & Calabretta, M.R. 2002, A&A, 395, 1061 (Paper I).

Morrison, J.E., & Stamper, B., "Image Distortions for MIPS (Without SED Mode)", MIPS instrument team memo, Feb 13, 2003.

6 Appendix: Converting between new and old schemes

A non-singular 2-dimensional CD matrix may be deconstructed into separate steps of scaling, skew, and rotation, applied in that order. This parametrization is presented in Morrison & Stamper (2003). Let s_1 and s_2 represent pixel scales in units of degrees per pixel for u and v , respectively, at the origin. We constrain s_2 to be positive, and let s_1 be positive or negative as needed. (For MIPS imaging mode BCDs and IRAC BCDs, s_1 will be negative.) The pixel axes at the u, v origin are first scaled (and will flip the u axis for negative s_1). Define β to be the skew angle, that is, the departure from perpendicularity between the scaled axis 1 and scaled axis 2, with the sense that positive β means these scaled axes are more than 90 degrees apart when mapped onto the plane of projection. Finally, let ρ be the angle from scaled axis 2 to North, with the positive sense defined by the North-to-East direction. The angles are depicted graphically in Figure 2.

The steps of scaling, skew, and rotation may be written as

$$\begin{pmatrix} CD_{11} & CD_{12} \\ CD_{21} & CD_{22} \end{pmatrix} = \begin{pmatrix} \cos(\rho) & -\sin(\rho) \\ \sin(\rho) & \cos(\rho) \end{pmatrix} \begin{pmatrix} \sec(\beta) & 0 \\ -\tan(\beta) & 1 \end{pmatrix} \begin{pmatrix} s_1 & 0 \\ 0 & s_2 \end{pmatrix} \quad (4)$$

The CD-matrix may be equivalently recast in the form

$$\begin{pmatrix} CD_{11} & CD_{12} \\ CD_{21} & CD_{22} \end{pmatrix} = \begin{pmatrix} s_1 \cos(\rho) & -s_2 \sin(\rho) \\ s_1 \sin(\rho) & s_2 \cos(\rho) \end{pmatrix} \begin{pmatrix} \sec(\beta) & 0 \\ -\left(\frac{s_1}{s_2}\right) \tan(\beta) & 1 \end{pmatrix} \quad (5)$$

The steps to derive rotation, scale and skew follow.

First, find ρ using an $\arctan2(y,x)$ function that finds the proper quadrant from two arguments (cf. the “arg” function in Paper II):

$$\rho = \text{ATAN2}(-CD_{12}, CD_{22}) \quad (6)$$

Second, form a new CD matrix, called rCD here, by rotating the original:

$$rCD = \begin{pmatrix} \cos(\rho) & \sin(\rho) \\ -\sin(\rho) & \cos(\rho) \end{pmatrix} \times CD \quad (7)$$

Use the elements of rCD to find the scales and skew.

$$s_2 = rCD_{22} \quad (8)$$

$$\beta = \sin^{-1} \left(-\frac{rCD_{21}}{rCD_{11}} \right) \quad (9)$$

$$s_1 = rCD_{11} * \cos(\beta) \quad (10)$$

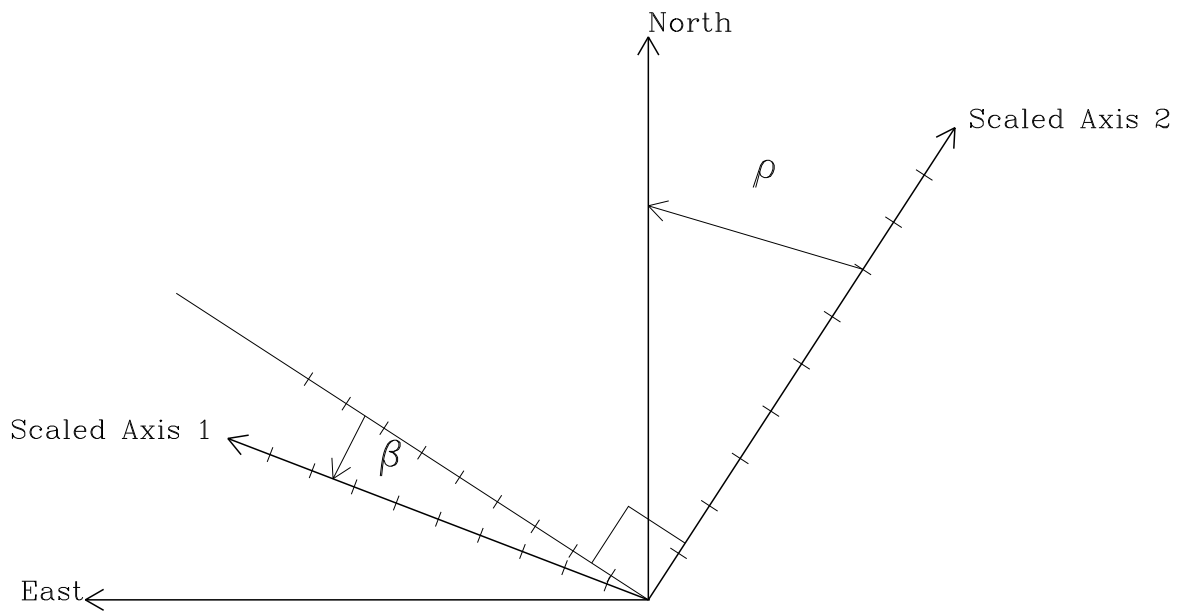


Figure 2: Schematic figure showing scaled pixel axes for the reference location (CRPIX1, CRPIX2) mapped onto the plane-of-projection. Angle β is the skew, exaggerated here. Angle ρ gives the orientation of Scaled Axis 2 with respect to the plane of projection. This figure assumes $s_1 < 0$ and $s_2 > 0$, as for MIPS imaging mode BCDs and IRAC BCDs.

CDELTA1 = s_1 , CDELTA2 = s_2 , and CROTA2 = ρ .

The distortion coefficients must also be transformed. From Equations 1 and 5, the skew term can be multiplied into $u + f(u, v)$ and $v + g(u, v)$ (as in Morrison & Stamper). Denoting the CROTA2+CDELTA distortion coefficients with lower-case letters, and the original CD-matrix distortion coefficients with upper-case letters as in Equations 2 and 3, the translations are

$$a_{10} = \sec(\beta) - 1, \tag{11}$$

$$a_{01} = 0, \tag{12}$$

$$b_{10} = -(s_1/s_2) \tan(\beta), \tag{13}$$

$$b_{01} = 0, \tag{14}$$

$$a_{pq} = \sec(\beta)A_{pq}, p + q \geq 2 \tag{15}$$

$$b_{pq} = B_{pq} - \frac{s_1}{s_2} \tan(\beta)A_{pq}, p + q \geq 2. \tag{16}$$

The reverse polynomials are much more difficult to transform. Since these exist only for speedier inversion, a reasonable approach is to generate points using the forward distortion coefficients, and do a least-squares fit to get reverse coefficients. For SIRTTF distortion polynomials, this can be accomplished by inverting a matrix equation. (Thanks to John Good of IRSA for pointing this out.)

We end this Appendix by noting that any arbitrary distortion polynomials for the pixels to sky direction, as long as they contain no constant terms, can easily be translated to the CD-matrix-plus-quadratic-and-higher form by multiplying out all the terms, collecting the linear ones into the CD matrix, and multiplying the higher-order terms by the inverse of that CD matrix. Then, the technique in the preceding paragraph may be used to find reverse coefficients.